

NON-MARKOVIAN QUANTUM STOCHASTIC EQUATION FOR TWO COUPLED OSCILLATORS

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Abstract

The system of nonlinear Langevin equations was obtained by using Hamiltonian's operator of two coupling quantum oscillators which are interacting with heat bath. By using the analytical solution of these equations, the analytical expressions for transport coefficients was found. Generalized Langevin equations and fluctuation-dissipation relations are derived for the case of a nonlinear non-Markovian noise. The explicit expressions for the time-dependent friction and diffusion coefficients are presented for the case of linear couplings in the coordinate between the collective two coupled harmonic oscillators and heat bath.

Keywords. Open quantum systems; Heat bath; Friction and Diffusion coefficients; Non-Markovian dynamics.

I. INTRODUCTION

Nowadays, one of the intensively developing topics of theoretical and mathematical physics is the non-equilibrium quantum theory. The study of the dynamics of open systems is directed towards derivation of transport equations and finding transport coefficients which they include. Many works are devoted to developing of formalism for the description of statistical and dynamical behavior of open systems. Powerful apparatus for solving complicated statistical problems of open systems is the theory of Markovian random processes and diffusion type processes, which has the origin of Brownian motion. However, the use of models of Markovian random process in many cases is quite rough, and in some cases - actually inapplicable. In that reason, designing of mathematical methods to consider non-Markovian random processes becomes natural and realistic. One of the possibilities of mathematical representation of non-Markovian process is using of integro-differential equations rather than differential equations. This kind of approach in essence allows to take into account the memory of the system when random process exists in it. An important problem in the theory of quantum open systems is the study of reduced (i.e. averaged over the reservoir state) dynamics of the system. In this case we usually suppose that reservoir is in equilibrium state. Reduced dynamics is described by the master equations for reduced system density matrix or for time evolution of system's observables averaged over the reservoir's state. Exact master equations include effects of memory and are complicated for practical study. Study of behaviors of dissipative quantum non-Markovian system beyond weak coupling or high temperatures draws an interest into exact solvable models [1–9]. In these models the internal subsystem (i.e., reservoir) is represented by a set of harmonic oscillators, whose interaction with a collective subsystem of harmonic oscillators is realized by the linear coupling between coordinates. Density of oscillators and coupling constants between internal and collective subsystems are chosen so that equations of motion for averages to be consistent with the classical formalism. Among quantum transport equations one can recommend the phenomenological Lindblad equation [10]. This is a deterministic equation, which can be obtained by averaging of stochastic Langevin equation by the controlling quantum noise. In kinetic theory, Langevin's method significantly simplifies the calculation of non-equilibrium quantum and thermal fluctuations and provides a clear description of both Markovian and non-Markovian dynamics of the process. The description below is devoted

to the elegant method to obtain non-stationary friction and diffusion coefficients for subsystem in case of arbitrary damping temperature. The transport coefficients also includes non-Markovian effects. As a starting point, Langevin approach is used, which is widely used for considering fluctuation and dissipation effects in macroscopic systems.

A quantum oscillator coupled to a heat bath is a very important and useful problem for many processes dealing with dynamics of open quantum systems [11]. In this work the problem of two coupled quantum oscillators interacting with ensembles of harmonic oscillators is considered.

II. GENERALIZED NON-MARKOVIAN QUANTUM LANGEVIN EQUATIONS

Let us define the microscopic Hamiltonian H of the total system (internal subsystem plus collective subsystem), which will be used to obtain non-Markovian quantum stochastic Langevin equations and time-dependent transport coefficients for the collective subsystem. In a quantum Hamiltonian was constructed for the systems, which is explicitly dependent on the collective coordinates R and β , canonically conjugate collective momentums P and P_β and internal degrees of freedom

$$H = H_s + H_b + H_{sb} \quad (1)$$

$$\begin{aligned} H_s &= \frac{P^2}{2m_1} + \eta \frac{m_1 \omega_1^2 R^2}{2} + \frac{P_\beta^2}{2m_2} + \frac{m_2 \omega_2^2 \beta^2}{2} + g_{R\beta} (R \cdot \beta) \\ H_b &= \sum_\nu \hbar \omega_\nu b_\nu^\dagger b_\nu \\ H_{sb} &= \sum_\nu (\alpha_\nu R + g_\nu \beta) (b_\nu^\dagger + b_\nu) \end{aligned} \quad (2)$$

where $\eta = \pm 1$. The coupling parameters α_ν and g_ν are

$$\alpha_\nu^2 = \frac{\lambda_1 \Gamma_\nu^2}{\hbar}, \quad g_\nu^2 = \frac{\lambda_2 \Gamma_\nu^2}{\hbar} \quad (3)$$

where λ_1 and λ_2 are parameters which measure the average strengths of the interactions and Γ_ν are the coupling constants. b_ν^\dagger and b_ν are the phonon production and annihilation operators that describe internal excitations of the system with energy $\hbar \omega_\nu$. For the sake of simplicity, we omit the signs of the operators. The quantities H_s and H_b are the Hamiltonians of the collective and the internal subsystem respectively. The quantity H_{sb} describes coupling

of the collective motion with the internal excitations and is a source of dissipative terms appearing in the equations for the operators of the collective variables.

Using Hamiltonian (1), we obtain a system of quantum Heisenberg equations for the operators related to the collective and internal motion

$$\begin{aligned}
\dot{R}(t) &= \frac{i}{\hbar}[H, R] = \frac{P(t)}{m_1} \\
\dot{\beta}(t) &= \frac{i}{\hbar}[H, \beta] = \frac{P_\beta(t)}{m_2} \\
\dot{P}(t) &= \frac{i}{\hbar}[H, P] = -\eta m_1 \omega_1^2 R(t) - \sum_\nu \alpha_\nu (b_\nu^\dagger(t) + b_\nu(t)) - g_{R\beta} \beta(t) \\
\dot{P}_\beta(t) &= \frac{i}{\hbar}[H, P_\beta] = -m_2 \omega_2^2 \beta(t) - \sum_\nu g_\nu (b_\nu^\dagger(t) + b_\nu(t)) - g_{R\beta} R(t)
\end{aligned} \tag{4}$$

and

$$\begin{aligned}
\dot{b}_\nu^\dagger(t) &= \frac{i}{\hbar}[H, b_\nu^\dagger] = i\omega_\nu b_\nu^\dagger(t) + \frac{i}{\hbar} [\alpha_\nu R(t) + g_\nu \beta(t)], \\
\dot{b}_\nu(t) &= \frac{i}{\hbar}[H, b_\nu] = -i\omega_\nu b_\nu(t) - \frac{i}{\hbar} [\alpha_\nu R(t) + g_\nu \beta(t)]
\end{aligned} \tag{5}$$

The solutions of Eqs.(5) are

$$\begin{aligned}
b_\nu^\dagger(t) &= f_\nu^\dagger(t) - \frac{1}{\hbar\omega_\nu} (\alpha_\nu R(t) + g_\nu \beta(t)) + \frac{1}{\omega_\nu} \int_0^t d\tau e^{i\omega_\nu(t-\tau)} [\alpha_\nu \dot{R}(t) + g_\nu \dot{\beta}(t)], \\
b_\nu(t) &= f_\nu(t) - \frac{1}{\hbar\omega_\nu} (\alpha_\nu R(t) + g_\nu \beta(t)) + \frac{1}{\omega_\nu} \int_0^t d\tau e^{-i\omega_\nu(t-\tau)} [\alpha_\nu \dot{R}(t) + g_\nu \dot{\beta}(t)]
\end{aligned} \tag{6}$$

Therefore,

$$b_\nu^\dagger(t) + b_\nu(t) = f_\nu^\dagger(t) + f_\nu(t) - \frac{2}{\hbar\omega_\nu} (\alpha_\nu R(t) + g_\nu \beta(t)) \tag{7}$$

$$+ \frac{2}{\hbar\omega_\nu} \int_0^t d\tau [\alpha_\nu \dot{R}(t) + g_\nu \dot{\beta}(t)] \cos(\omega_\nu(t-\tau)) \tag{8}$$

where

$$\begin{aligned}
f_\nu^\dagger(t) &= \left[b_\nu^\dagger(0) + \frac{\alpha_\nu R(0)}{\hbar\omega_\nu} + \frac{g_\nu \beta(0)}{\hbar\omega_\nu} \right] e^{i\omega_\nu t} \\
f_\nu(t) &= \left[b_\nu(0) + \frac{\alpha_\nu R(0)}{\hbar\omega_\nu} + \frac{g_\nu \beta(0)}{\hbar\omega_\nu} \right] e^{-i\omega_\nu t}
\end{aligned} \tag{9}$$

Substituting Eq.(8) into Eqs.(4), we eliminate the bath variables from the equations of motion of the collective subsystem and obtain the nonlinear integro-differential stochastic

dissipative equations

$$\begin{aligned}
\dot{R}(t) &= \frac{P(t)}{m_1} \\
\dot{\beta}(t) &= \frac{P_\beta(t)}{m_2} \\
\dot{P}(t) &= -\eta m_1 \omega_1^2 R(t) - g_{R\beta} \beta(t) - F_1(t) + 2 \sum_\nu \frac{1}{\hbar \omega_\nu} \left(\alpha_\nu^2 R(t) + \alpha_\nu g_\nu \beta(t) \right) - \\
&\quad - 2 \sum_\nu \frac{1}{\hbar \omega_\nu} \int_0^t d\tau \left[\alpha_\nu \dot{R}(\tau) + \alpha_\nu g_\nu \dot{\beta}(\tau) \right] \cos(\omega_\nu(t - \tau)) \\
\dot{P}_\beta(t) &= -m_2 \omega_2^2 \beta(t) - g_{R\beta} R(t) - F_2(t) + 2 \sum_\nu \frac{1}{\hbar \omega_\nu} \left(\alpha_\nu g_\nu R(t) + g_\nu^2 \beta(t) \right) - \\
&\quad - 2 \sum_\nu \frac{1}{\hbar \omega_\nu} \int_0^t d\tau \left[\alpha_\nu g_\nu \dot{R}(\tau) + g_\nu \dot{\beta}(\tau) \right] \cos(\omega_\nu(t - \tau))
\end{aligned} \tag{10}$$

The presence of the integral parts in these equations indicates the non-Markovian character of the system. Since in comparison with Refs.[7, 9] we do not introduce the counter-term in the Hamiltonian, the stiffnesses of the potentials are renormalized in the equations above. Due to the operators

$$\begin{aligned}
F_1(t) &= \sum_\nu F_\alpha^\nu(t) = \sum_\nu \alpha_\nu \left(f_\nu^\dagger(t) + f_\nu(t) \right) \\
F_2(t) &= \sum_\nu F_g^\nu(t) = \sum_\nu g_\nu \left(f_\nu^\dagger(t) + f_\nu(t) \right)
\end{aligned}$$

which play the role of random forces in the coordinates, Eqs.(10) can be called the generalized nonlinear quantum Langevin equations. Following the usual procedure of statistical mechanics, we identify these operators as fluctuations because of the uncertainty in the initial conditions for the bath operators.

$$\begin{aligned}
\dot{R}(t) &= \frac{P(t)}{m_1} \\
\dot{\beta}(t) &= \frac{P_\beta(t)}{m_2} \\
\dot{P}(t) &= -\left(\eta m_1 \omega_1^2 - \Delta_1 \right) R(t) - (g_{R\beta} - \Delta_2) \beta(t) - F_1(t) - \frac{1}{m_1} \int_0^t d\tau K_1(t - \tau) P(\tau) - \\
&\quad - \frac{1}{m_2} \int_0^t d\tau K_2(t - \tau) P_\beta(\tau) \\
\dot{P}_\beta(t) &= -(g_{R\beta} - \Delta_3) R(t) - \left(m_2 \omega_2^2 - \Delta_4 \right) \beta(t) - F_2(t) - \frac{1}{m_1} \int_0^t d\tau K_3(t - \tau) P(\tau) -
\end{aligned}$$

$$- \frac{1}{m_2} \int_0^t d\tau K_4(t - \tau) P_\beta(\tau) \quad (11)$$

where

$$\Delta_1 = \sum_\nu \frac{2\alpha_\nu^2}{\hbar\omega_\nu}, \quad \Delta_2 = \Delta_3 = \sum_\nu \frac{2\alpha_\nu g_\nu}{\hbar\omega_\nu}, \quad \Delta_4 = \sum_\nu \frac{2g_\nu^2}{\hbar\omega_\nu}$$

$$K_1(t - \tau) = \sum_\nu \frac{2\alpha_\nu^2}{\hbar\omega_\nu} \cos(\omega_\nu(t - \tau))$$

$$K_2(t - \tau) = K_3(t - \tau) = \sum_\nu \frac{2\alpha_\nu g_\nu}{\hbar\omega_\nu} \cos(\omega_\nu(t - \tau))$$

$$K_4(t - \tau) = \sum_\nu \frac{2g_\nu^2}{\hbar\omega_\nu} \cos(\omega_\nu(t - \tau))$$

In equations of motion (11), the dissipative kernels $K_1(t - \tau)$, $K_2(t - \tau)$, $K_3(t - \tau)$ and $K_4(t - \tau)$ are separated in the terms proportional to \dot{R} , $\dot{\beta}$ and \dot{P} , \dot{P}_β [8, 12, 13]. These kernels depend on the coefficients of H_{sb} . Since the dissipative kernels do not depend on the number of phonons, they do not depend on the bath temperature T either. The temperature and the fluctuation enter into the consideration of the dynamics of R , β and P , P_β via the distributions of the initial conditions for the internal system. The explicit expressions for the dissipative kernels $K_1(t - \tau)$, $K_2(t - \tau)$, $K_3(t - \tau)$ and $K_4(t - \tau)$ and for the operators $F_1(t) = \sum_\nu F_\alpha^\nu(t)$ and $F_2(t) = \sum_\nu F_g^\nu(t)$ in (11), which play the role of the random P and P_β , forces, were obtained in [1].

In statistical physics the operators $F_\alpha^\nu(t)$ and $F_g^\nu(t)$ are identified as usual with the fluctuations due to the uncertainty of the initial conditions for the bath operators. To determine statistical properties of these fluctuations, we consider an ensemble of initial conditions in which $R(0)$, $\beta(0)$, $P(0)$ and $P_\beta(0)$ are given, and the initial bath operators are chosen from the canonical ensemble [8, 12, 13]. In this ensemble the fluctuations $F_\alpha^\nu(t)$ and $F_g^\nu(t)$ are Gaussian distributions and have zero mean values

$$\ll F_\alpha^\nu(t) \gg = \ll F_g^\nu(t) \gg = 0 \quad (12)$$

and nonzero second moments. The symbol $\ll \dots \gg$ denotes the average over the bath variables. The Gaussian distribution of the random forces corresponds to the case where the bath is a set of harmonic oscillators [9, 14, 15]. To calculate the correlation functions of

the fluctuations, we will use the bath with the Bose-Einstein statistics

$$\begin{aligned}
\ll f_\nu^+(t) f_{\nu'}^+(t') \gg &= \ll f_\nu(t) f_{\nu'}(t') \gg = 0, \\
\ll f_\nu^+(t) f_{\nu'}(t') \gg &= \delta_{\nu,\nu'} n_\nu e^{i\omega_\nu(t-t')}, \\
\ll f_\nu(t) f_{\nu'}^+(t') \gg &= \delta_{\nu,\nu'} (n_\nu + 1) e^{-i\omega_\nu(t-t')},
\end{aligned} \tag{13}$$

where $n_\nu = [\exp(\hbar\omega_\nu/T) - 1]^{-1}$ are the temperature occupation numbers for phonons.

Thus, a system of generalized nonlinear Langevin equations (11) is obtained. The presence of the integral terms in the equations of motion means that the non-Markovian system remembers the motion over the trajectory prior to the time t . Analytical solution is possible if the functionals in (11) are replaced by their mean values considered to be weakly varying in time t and the renormalized potential is approximated by the harmonic (or inverted) oscillator. In this case, we have a system of generalized Langevin equations with dissipative memory kernels. We will solve them using the Laplace transform \mathcal{L} to obtain linear equations for images.

$$\begin{aligned}
sR(s) - \frac{P(s)}{m_1} &= R(0) \\
s\beta(s) - \frac{P_\beta(s)}{m_2} &= \beta(0) \\
(\eta m_1 \omega_1^2 - \Delta_1) R(s) + (g_{R\beta} - \Delta_2) \beta(s) + \left(s + \frac{1}{m_1} K_1(s)\right) P(s) \\
+ \frac{1}{m_2} K_2(s) P_\beta(s) &= P(0) - F_1(s) \\
(g_{R\beta} - \Delta_3) R(s) + (m_2 \omega_2^2 - \Delta_4) \beta(s) + \frac{1}{m_1} K_3(s) P(s) \\
+ \left(s + \frac{1}{m_2} K_4(s)\right) P_\beta(s) &= P_\beta(0) - F_2(s)
\end{aligned} \tag{14}$$

The above originals can be found using the residue theorem, and the solutions $R(t)$, $\beta(t)$, $P(t)$ and $P_\beta(t)$ can be written down in terms of the roots s_i of the equation

$$\begin{aligned}
d(s) = & s^2 \left(s + \frac{1}{m_1} K_1(s)\right) \left(s + \frac{1}{m_2} K_4(s)\right) - \frac{1}{m_1 m_2} s^2 K_2(s) K_3(s) + \\
& + \frac{s}{m_1} \left[(\eta m_1 \omega_1^2 - \Delta_1) \left(s + \frac{1}{m_2} K_4(s)\right) - (g_{R\beta} - \Delta_3) \frac{1}{m_2} K_2(s) \right] + \\
& + \frac{s}{m_2} \left[(m_2 \omega_2^2 - \Delta_4) \left(s + \frac{1}{m_1} K_1(s)\right) - (g_{R\beta} - \Delta_2) \frac{1}{m_1} K_3(s) \right] + \\
& + \frac{1}{m_1 m_2} [(\eta m_1 \omega_1^2 - \Delta_1) (m_2 \omega_2^2 - \Delta_4) - (g_{R\beta} - \Delta_2) (g_{R\beta} - \Delta_3)] = 0
\end{aligned} \tag{15}$$

Expressions for the images yield explicit expressions for the originals

$$\begin{aligned}
R(t) &= A_1(t)R(0) + A_2(t)\beta(0) + A_3(t)P(0) + A_4(t)P_\beta(0) - I_1(t) - I'_1(t) \\
\beta(t) &= B_1(t)R(0) + B_2(t)\beta(0) + B_3(t)P(0) + B_4(t)P_\beta(0) - I_2(t) - I'_2(t) \\
P(t) &= C_1(t)R(0) + C_2(t)\beta(0) + C_3(t)P(0) + C_4(t)P_\beta(0) - I_3(t) - I'_3(t) \\
P_\beta(t) &= D_1(t)R(0) + D_2(t)\beta(0) + D_3(t)P(0) + D_4(t)P_\beta(0) - I_4(t) - I'_4(t)
\end{aligned} \tag{16}$$

where

$$\begin{aligned}
I_R(t) &= \int_0^t A_3(\tau)F_1(t-\tau)d\tau; I'_R(t) = \int_0^t A_4(\tau)F_2(t-\tau)d\tau; \\
I_\beta(t) &= \int_0^t B_3(\tau)F_1(t-\tau)d\tau; I'_\beta(t) = \int_0^t B_4(\tau)F_2(t-\tau)d\tau; \\
I_P(t) &= \int_0^t C_3(\tau)F_1(t-\tau)d\tau; I'_P(t) = \int_0^t C_4(\tau)F_2(t-\tau)d\tau; \\
I_{P_\beta}(t) &= \int_0^t D_3(\tau)F_1(t-\tau)d\tau; I'_{P_\beta}(t) = \int_0^t D_4(\tau)F_2(t-\tau)d\tau;
\end{aligned} \tag{17}$$

where the coefficients are defined as

$$\begin{aligned}
A_1(t) &= \mathcal{L}^{-1} \left[\frac{1}{d(s)} \left(s \left(s + \frac{1}{m_1} K_1(s) \right) \left(s + \frac{1}{m_2} K_4(s) \right) + \frac{1}{m_2} (m_2 \omega_2^2 - \Delta_4) \left(s + \frac{1}{m_1} K_1(s) \right) - \right. \right. \\
&\quad \left. \left. - \frac{1}{m_1 m_2} K_3(s) (g_{R\beta} - \Delta_2 + s K_2(s)) \right) \right] \\
A_2(t) &= \mathcal{L}^{-1} \left[\frac{1}{d(s)} \left(\frac{1}{m_1 m_2} (m_2 \omega_2^2 - \Delta_4) K_2(s) - \frac{1}{m_1} (g_{R\beta} - \Delta_2) \left(s + \frac{1}{m_2} K_4(s) \right) \right) \right] \\
A_3(t) &= \mathcal{L}^{-1} \left[\frac{1}{d(s)} \left(\frac{1}{m_1 m_2} (m_2 \omega_2^2 - \Delta_4) + \frac{1}{m_1} s \left(s + \frac{1}{m_2} K_4(s) \right) \right) \right] \\
A_4(t) &= -\mathcal{L}^{-1} \left[\frac{1}{d(s)} \left(\frac{1}{m_1 m_2} (g_{R\beta} - \Delta_2) + \frac{s}{m_1 m_2} K_2(s) \right) \right] \\
B_1(t) &= \mathcal{L}^{-1} \left[\frac{1}{d(s)} \left(\frac{1}{m_1 m_2} (\eta m_1 \omega_1^2 - \Delta_1) K_3(s) - \frac{1}{m_2} (g_{R\beta} - \Delta_3) \left(s + \frac{1}{m_1} K_1(s) \right) \right) \right] \\
B_2(t) &= \mathcal{L}^{-1} \left[\frac{1}{d(s)} \left(s \left(s + \frac{1}{m_1} K_1(s) \right) \left(s + \frac{1}{m_2} K_4(s) \right) + \frac{1}{m_1} (\eta m_1 \omega_1^2 - \Delta_1) \left(s + \frac{1}{m_2} K_4(s) \right) - \right. \right. \\
&\quad \left. \left. - \frac{1}{m_1 m_2} K_2(s) (g_{R\beta} - \Delta_3 + s K_3(s)) \right) \right] \\
B_3(t) &= -\mathcal{L}^{-1} \left[\frac{1}{d(s)} \left(\frac{1}{m_1 m_2} (g_{R\beta} - \Delta_3) + \frac{s}{m_1 m_2} K_3(s) \right) \right] \\
B_4(t) &= \mathcal{L}^{-1} \left[\frac{1}{d(s)} \left(\frac{1}{m_1 m_2} (\eta m_1 \omega_1^2 - \Delta_1) + \frac{1}{m_2} s \left(s + \frac{1}{m_1} K_1(s) \right) \right) \right]
\end{aligned}$$

$$\begin{aligned}
C_1(t) &= \mathcal{L}^{-1} \left[\frac{1}{d(s)} \left(\frac{s}{m_2} (g_{R\beta} - \Delta_3) K_2(s) - s (\eta m_1 \omega_1^2 - \Delta_1) \left(s + \frac{1}{m_2} K_4(s) \right) + \right. \right. \\
&\quad \left. \left. + \frac{1}{m_2} (g_{R\beta} - \Delta_2) (g_{R\beta} - \Delta_3) - \frac{1}{m_2} (\eta m_1 \omega_1^2 - \Delta_1) (m_2 \omega_2^2 - \Delta_4) \right) \right] \\
C_2(t) &= \mathcal{L}^{-1} \left[\frac{1}{d(s)} \left(\frac{s}{m_2} (m_2 \omega_2^2 - \Delta_4) K_2(s) - s (g_{R\beta} - \Delta_2) \left(s + \frac{1}{m_2} K_4(s) \right) \right) \right] \\
C_3(t) &= \mathcal{L}^{-1} \left[\frac{1}{d(s)} \left(s^2 \left(s + \frac{1}{m_2} K_4(s) \right) + \frac{s}{m_2} (m_2 \omega_2^2 - \Delta_4) \right) \right] \\
C_4(t) &= -\mathcal{L}^{-1} \left[\frac{1}{d(s)} \left(\frac{s^2}{m_2} K_2(s) + \frac{s}{m_2} (g_{R\beta} - \Delta_2) \right) \right] \\
D_1(t) &= \mathcal{L}^{-1} \left[\frac{1}{d(s)} \left(\frac{s}{m_1} (\eta m_1 \omega_1^2 - \Delta_1) K_3(s) - s (g_{R\beta} - \Delta_3) \left(s + \frac{1}{m_1} K_1(s) \right) \right) \right] \\
D_2(t) &= \mathcal{L}^{-1} \left[\frac{1}{d(s)} \left(\frac{s}{m_1} (g_{R\beta} - \Delta_2) K_3(s) - s (m_2 \omega_2^2 - \Delta_4) \left(s + \frac{1}{m_1} K_1(s) \right) + \right. \right. \\
&\quad \left. \left. + \frac{1}{m_1} (g_{R\beta} - \Delta_2) (g_{R\beta} - \Delta_3) - \frac{1}{m_1} (\eta m_1 \omega_1^2 - \Delta_1) (m_2 \omega_2^2 - \Delta_4) \right) \right] \\
D_3(t) &= -\mathcal{L}^{-1} \left[\frac{1}{d(s)} \left(\frac{s^2}{m_1} K_3(s) + \frac{s}{m_1} (g_{R\beta} - \Delta_3) \right) \right] \\
D_4(t) &= \mathcal{L}^{-1} \left[\frac{1}{d(s)} \left(s^2 \left(s + \frac{1}{m_1} K_1(s) \right) + \frac{s}{m_1} (\eta m_1 \omega_1^2 - \Delta_1) \right) \right]
\end{aligned}$$

Here \mathcal{L}^{-1} denotes the inverse Laplace transform, and $K_1(s)$, $K_2(s)$, $K_3(s)$, and $K_4(s)$ are the Laplace images of the dissipative kernels.

It is convenient to introduce the spectral density $D(\omega)$ of the heat bath excitations which allows us to replace the sum over different oscillators ν by the integral over the frequency: $\sum_{\nu} \dots \rightarrow \int_0^{\infty} d\omega D(\omega) \dots$. This replacement is accompanied by the following replacements: $\Gamma_{\nu} \rightarrow \Gamma_{\omega}$, $\omega_{\nu} \rightarrow \omega$ and $n_{\nu} \rightarrow n_{\omega}$. Let us consider the following spectral functions

$$D(\omega) \frac{|\Gamma(\omega)|^2}{\hbar^2 \omega} = \frac{1}{\pi} \frac{\gamma^2}{\gamma^2 + \omega^2}$$

where the memory time γ^{-1} of the dissipation is inverse to the phonon bandwidth of the heat bath excitations which are coupled with the collective oscillator. If we rewrite the sum \sum_{ν} as the integral over the bath frequencies with the density of states, we obtain

$$K_1(t) = \lambda_1 \gamma e^{-\gamma|t|}, \quad K_2(t) = K_3(t) = \lambda_1^{1/2} \lambda_2^{1/2} \gamma e^{-\gamma|t|}, \quad K_4(t) = \lambda_2 \gamma e^{-\gamma|t|}$$

and

$$\Delta_1 = \lambda_1 \gamma, \quad \Delta_2 = \Delta_3 = \lambda_1^{1/2} \gamma_2^{1/2} \gamma, \quad \Delta_4 = \lambda_2 \gamma$$

We assume that there are no correlations between $F_1(t)$ and $F_2(t)$, so that

$$\sum_{\nu} \frac{\alpha_{\nu} g_{\nu}}{\hbar \omega_{\nu}} \equiv 0 \quad (18)$$

The dissipative kernels are $K_2(s) = K_3(s) \equiv 0$ and $\Delta_2 = \Delta_3 \equiv 0$.

$$K_1(s) = \frac{\lambda_1 \gamma}{(s + \gamma)}, K_4(s) = \frac{\lambda_2 \gamma}{(s + \gamma)} \quad (19)$$

So, in this case, the solutions for the collective variables (16) include the following time-dependent coefficients:

$$\begin{aligned} A_1(t) &= \frac{1}{m_1 m_2} \sum_{i=1}^6 \xi_i \left[m_1 s_i (s_i + \gamma) \left(m_2 (s_i + \gamma) (s_i^2 + \omega_2^2) - \lambda_2 \gamma^2 \right) + \right. \\ &\quad \left. + \lambda_1 \gamma \left(m_2 (s_i + \gamma) (s_i^2 + \omega_2^2) - \lambda_2 \gamma^2 \right) \right] e^{s_i t} \\ A_2(t) &= -\frac{g_{R\beta}}{m_1 m_2} \sum_{i=1}^6 \xi_i \left[m_2 s_i (s_i + \gamma)^2 + \lambda_2 \gamma (s_i + \gamma) \right] e^{s_i t} \\ A_3(t) &= \frac{1}{m_1 m_2} \sum_{i=1}^6 \xi_i (s_i + \gamma) \left[m_2 (s_i + \gamma) (s_i^2 + \omega_2^2) - \lambda_2 \gamma^2 \right] e^{s_i t} \\ A_4(t) &= -\frac{g_{R\beta}}{m_1 m_2} \sum_{i=1}^6 \xi_i (s_i + \gamma)^2 e^{s_i t} \\ B_1(t) &= -\frac{g_{R\beta}}{m_1 m_2} \sum_{i=1}^6 \xi_i (s_i + \gamma) \left[m_1 s_i (s_i + \gamma) + \lambda_1 \gamma \right] e^{s_i t} \\ B_2(t) &= \frac{1}{m_1 m_2} \sum_{i=1}^6 \xi_i \left[m_2 s_i (s_i + \gamma) \left(m_1 (s_i + \gamma) (s_i^2 + \eta \omega_1^2) - \lambda_1 \gamma^2 \right) + \right. \\ &\quad \left. + \lambda_2 \gamma \left(m_1 (s_i + \gamma) (s_i^2 + \eta \omega_1^2) - \lambda_1 \gamma^2 \right) \right] e^{s_i t} \\ B_3(t) &= -\frac{g_{R\beta}}{m_1 m_2} \sum_{i=1}^6 \xi_i (s_i + \gamma)^2 e^{s_i t} \\ B_4(t) &= \frac{1}{m_1 m_2} \sum_{i=1}^6 \xi_i (s_i + \gamma) \left[m_1 (s_i + \gamma) (s_i^2 + \eta \omega_1^2) - \lambda_1 \gamma^2 \right] e^{s_i t} \\ C_1(t) &= \frac{1}{m_2} \sum_{i=1}^6 \xi_i (s_i + \gamma) \left[g_{R\beta}^2 (s_i + \gamma) - \eta m_1 \omega_1^2 \left(m_2 (s_i + \gamma) (s_i^2 + \omega_2^2) - \lambda_2 \gamma^2 \right) + \right. \\ &\quad \left. + \lambda_1 \gamma \left(m_2 (s_i + \gamma) (s_i^2 + \omega_2^2) - \lambda_2 \gamma^2 \right) \right] e^{s_i t} \\ C_2(t) &= -\frac{g_{R\beta}}{m_2} \sum_{i=1}^6 \xi_i s_i (s_i + \gamma) \left(m_2 s_i (s_i + \gamma) + \lambda_2 \gamma \right) e^{s_i t} \\ C_3(t) &= \frac{1}{m_2} \sum_{i=1}^6 \xi_i s_i (s_i + \gamma) \left(m_2 (s_i + \gamma) (s_i^2 + \omega_2^2) - \lambda_2 \gamma \right) e^{s_i t} \\ C_4(t) &= -\frac{g_{R\beta}}{m_2} \sum_{i=1}^6 \xi_i s_i (s_i + \gamma)^2 e^{s_i t} \end{aligned}$$

$$\begin{aligned}
D_1(t) &= -\frac{g_{R\beta}}{m_1} \sum_{i=1}^6 \xi_i s_i (s_i + \gamma) (m_1 s_i (s_i + \gamma) + \lambda_1 \gamma) e^{s_i t} \\
D_2(t) &= \frac{1}{m_1} \sum_{i=1}^6 \xi_i (s_i + \gamma) \left[g_{R\beta}^2 (s_i + \gamma) - m_2 \omega_2^2 \left(m_1 (s_i + \gamma) (s_i^2 + \eta \omega_1^2) - \lambda_1 \gamma^2 \right) + \right. \\
&\quad \left. + \lambda_2 \gamma \left(m_1 (s_i + \gamma) (s_i^2 + \eta \omega_1^2) - \lambda_1 \gamma^2 \right) \right] e^{s_i t} \\
D_3(t) &= -\frac{g_{R\beta}}{m_1} \sum_{i=1}^6 \xi_i s_i (s_i + \gamma)^2 e^{s_i t} \\
D_4(t) &= \frac{1}{m_1} \sum_{i=1}^6 \xi_i s_i (s_i + \gamma) \left(m_1 (s_i + \gamma) (s_i^2 + \eta \omega_1^2) - \lambda_1 \gamma \right) e^{s_i t}
\end{aligned}$$

Here, s_i are the roots of the following equation:

$$\frac{g_{R\beta}^2 (s_i + \gamma)^2}{m_1 m_2} - \left((s_i + \gamma) (s_i^2 + \eta \omega_1^2) - \frac{\lambda_1 \gamma^2}{m_1} \right) \left((s_i + \gamma) (s_i^2 + \omega_2^2) - \frac{\lambda_2 \gamma^2}{m_2} \right) = 0 \quad (20)$$

and $\xi_i = \left[\prod_{j \neq i} (s_i - s_j) \right]^{-1}$ with $i, j = 1 - 6$. These roots arise when we apply the residue theorem to perform integration in the inverse Laplace transformation.

A. Fluctuation-Dissipation Relations

An important relation between the dissipation in the dynamics of a system and the fluctuations in a heat bath with which the system interacts is the fluctuation-dissipation relation. A first example of its manifestation is the Nyquist noise in an electric circuit. This relation is of practical interest in the design of noisy systems. It is also of theoretical interest in statistical physics because it is a categorical relation which exists between the stochastic behavior of many microscopic particles and the deterministic behavior of a macroscopic system. It is therefore also useful for the description of the interaction of a system with fields, such as effects related to radiation reaction and vacuum fluctuations between atoms and fields in quantum optics. The form of the fluctuation-dissipation relation is usually given under near-equilibrium conditions via linear response theory. We will see in this paragraph that this relation has a much wider scope and a broader implication than has been understood before. In particular we want to apply this relations for problems involving dissipation kinetic energy the initial stage of heavy ions collisions.

In [1], fluctuation-dissipation relations were obtained for (11), which connect the macroscopic quantity that describes dissipation and the microscopic characteristic of the internal subsystem that expresses fluctuation of random forces. Validity of these relations means

that the dissipative kernels in the non-Markovian dynamic equations of motion are determined correctly. The quantum fluctuation-dissipation relation of this form was obtained in [8] and the references therein for the simple cases of the FC and RWA oscillators. Quantum fluctuation-dissipation relations differ from classical ones and are reduced to them in the limit of high temperature T (or when $\hbar \rightarrow 0$).

In addition to the temperature fluctuations, the quantum fluctuations are also considered in them. Since equations of motion (11) for the collective coordinates and momenta correspond to the fluctuation-dissipation relations, our formalism is the basis for describing quantum statistical effects of collective motion.

We obtain the following relationships for the symmetrized correlation functions ($k = \alpha, g$) of the random forces $\varphi_{kk'}^\nu(t, t') = \langle \langle F_k^\nu(t) F_{k'}^\nu(t') + F_{k'}^\nu(t') F_k^\nu(t) \rangle \rangle$:

$$\varphi_{kk'}^\nu(t, t') = 2k_\nu k_{\nu'} [2n_\nu + 1] \cos(\omega_\nu[t - t'])$$

Using the properties of random forces, we obtain the quantum fluctuation-dissipation relations

$$\begin{aligned} \sum_\nu \varphi_{\alpha\alpha}^\nu(t, t') \frac{\tanh\left[\frac{\hbar\omega_\nu}{2T}\right]}{\hbar\omega_\nu} &= K_1(t - t') \\ \sum_\nu \varphi_{\alpha g}^\nu(t, t') \frac{\tanh\left[\frac{\hbar\omega_\nu}{2T}\right]}{\hbar\omega_\nu} &= K_2(t - t') \\ \sum_\nu \varphi_{g\alpha}^\nu(t, t') \frac{\tanh\left[\frac{\hbar\omega_\nu}{2T}\right]}{\hbar\omega_\nu} &= K_3(t - t') \\ \sum_\nu \varphi_{gg}^\nu(t, t') \frac{\tanh\left[\frac{\hbar\omega_\nu}{2T}\right]}{\hbar\omega_\nu} &= K_4(t - t') \end{aligned} \quad (21)$$

The validity of the fluctuation-dissipation relationships means that we correctly specified the dissipative kernels in the non-Markovian equations of motion.

III. TRANSPORT COEFFICIENTS

In order to determine the transport coefficients, we use the solution (16). Averaging them over the whole system and taking the time derivative, we obtain the following system of equations for the first moments:

$$\langle \dot{R}(t) \rangle = \frac{\langle P(t) \rangle}{m_1}$$

$$\begin{aligned}
\langle \dot{\beta}(t) \rangle &= \frac{\langle P_\beta(t) \rangle}{m_2} \\
\langle \dot{P}(t) \rangle &= -\lambda_P \langle P(t) \rangle + \rho_R \langle P_\beta(t) \rangle - c_R \langle R(t) \rangle + \delta_R \langle \beta(t) \rangle \\
\langle \dot{P}_\beta(t) \rangle &= -\lambda_{P_\beta} \langle P_\beta(t) \rangle + \rho_\beta \langle P(t) \rangle - c_\beta \langle \beta(t) \rangle + \delta_\beta \langle R(t) \rangle
\end{aligned} \tag{22}$$

where the time-dependent coefficients $\lambda_P(t)$, $\lambda_{P_\beta}(t)$, $\rho_R(t)$, $\rho_\beta(t)$, $c_R(t)$, $c_\beta(t)$, $\delta_R(t)$, $\delta_\beta(t)$. The coefficients $\lambda_{P,P_\beta}(t)$ are related to the friction coefficients. The renormalized stiffnesses are $c_{R,\beta}(t)$. Using Eqs.(16), we write Eqs.(22) for the first moments in which the coefficients after simple algebra are

$$\begin{aligned}
\lambda_P(t) &= - \left\{ \left[B_1(t)\dot{C}_2(t) - B_2(t)\dot{C}_1(t) \right] [A_3(t)D_4(t) - A_4(t)D_3(t)] + \right. \\
&\quad + \left[B_1(t)\dot{C}_3(t) - B_3(t)\dot{C}_1(t) \right] [A_4(t)D_2(t) - A_2(t)D_4(t)] + \\
&\quad + \left[B_1(t)\dot{C}_4(t) - B_4(t)\dot{C}_1(t) \right] [A_2(t)D_3(t) - A_3(t)D_2(t)] + \\
&\quad + \left[B_2(t)\dot{C}_3(t) - B_3(t)\dot{C}_2(t) \right] [A_1(t)D_4(t) - A_4(t)D_1(t)] + \\
&\quad + \left[B_2(t)\dot{C}_4(t) - B_4(t)\dot{C}_2(t) \right] [A_3(t)D_1(t) - A_1(t)D_3(t)] + \\
&\quad \left. + \left[B_3(t)\dot{C}_4(t) - B_4(t)\dot{C}_3(t) \right] [A_1(t)D_2(t) - A_2(t)D_1(t)] \right\} / I(t) \\
\rho_R(t) &= \left\{ \left[C_1(t)\dot{C}_2(t) - C_2(t)\dot{C}_1(t) \right] [A_3(t)B_4(t) - A_4(t)B_3(t)] + \right. \\
&\quad + \left[C_1(t)\dot{C}_3(t) - C_3(t)\dot{C}_1(t) \right] [A_4(t)B_2(t) - A_2(t)B_4(t)] + \\
&\quad + \left[C_1(t)\dot{C}_4(t) - C_4(t)\dot{C}_1(t) \right] [A_2(t)B_3(t) - A_3(t)B_2(t)] + \\
&\quad + \left[C_2(t)\dot{C}_3(t) - C_3(t)\dot{C}_2(t) \right] [A_1(t)B_4(t) - A_4(t)B_1(t)] + \\
&\quad + \left[C_2(t)\dot{C}_4(t) - C_4(t)\dot{C}_2(t) \right] [A_3(t)B_1(t) - A_1(t)B_3(t)] + \\
&\quad \left. + \left[C_3(t)\dot{C}_4(t) - C_4(t)\dot{C}_3(t) \right] [A_1(t)B_2(t) - A_2(t)B_1(t)] \right\} / I(t) \\
c_R(t) &= - \left\{ \left[C_1(t)\dot{C}_2(t) - C_2(t)\dot{C}_1(t) \right] [B_3(t)D_4(t) - B_4(t)D_3(t)] + \right. \\
&\quad + \left[C_1(t)\dot{C}_3(t) - C_3(t)\dot{C}_1(t) \right] [B_4(t)D_2(t) - B_2(t)D_4(t)] + \\
&\quad + \left[C_1(t)\dot{C}_4(t) - C_4(t)\dot{C}_1(t) \right] [B_2(t)D_3(t) - B_3(t)D_2(t)] + \\
&\quad + \left[C_2(t)\dot{C}_3(t) - C_3(t)\dot{C}_2(t) \right] [B_1(t)D_4(t) - B_4(t)D_1(t)] + \\
&\quad + \left[C_2(t)\dot{C}_4(t) - C_4(t)\dot{C}_2(t) \right] [B_3(t)D_1(t) - B_1(t)D_3(t)] + \\
&\quad \left. + \left[C_3(t)\dot{C}_4(t) - C_4(t)\dot{C}_3(t) \right] [B_1(t)D_2(t) - B_2(t)D_1(t)] \right\} / I(t) \\
\delta_R(t) &= \left\{ \left[C_1(t)\dot{C}_2(t) - C_2(t)\dot{C}_1(t) \right] [A_4(t)D_3(t) - A_3(t)D_4(t)] + \right. \\
&\quad + \left[C_1(t)\dot{C}_3(t) - C_3(t)\dot{C}_1(t) \right] [A_2(t)D_4(t) - A_4(t)D_2(t)] + \\
&\quad \left. + \left[C_1(t)\dot{C}_4(t) - C_4(t)\dot{C}_1(t) \right] [A_3(t)D_2(t) - A_2(t)D_3(t)] + \right.
\end{aligned}$$

$$\begin{aligned}
& + [C_2(t)\dot{C}_3(t) - C_3(t)\dot{C}_2(t)] [A_4(t)D_1(t) - A_1(t)D_4(t)] + \\
& + [C_2(t)\dot{C}_4(t) - C_4(t)\dot{C}_2(t)] [A_1(t)D_3(t) - A_3(t)D_1(t)] + \\
& + [C_3(t)\dot{C}_4(t) - C_4(t)\dot{C}_3(t)] [A_2(t)D_1(t) - A_1(t)D_2(t)] \} / I(t) \\
I(t) & = [B_1(t)D_2(t) - B_2(t)D_1(t)] [A_4(t)C_3(t) - A_3(t)C_4(t)] + \\
& + [B_1(t)D_3(t) - B_3(t)D_1(t)] [A_2(t)C_4(t) - A_4(t)C_2(t)] + \\
& + [B_1(t)D_4(t) - B_4(t)D_1(t)] [A_3(t)C_2(t) - A_2(t)C_3(t)] + \\
& + [B_2(t)D_3(t) - B_3(t)D_2(t)] [A_4(t)C_1(t) - A_1(t)C_4(t)] + \\
& + [B_2(t)D_4(t) - B_4(t)D_2(t)] [A_1(t)C_3(t) - A_3(t)C_1(t)] + \\
& + [B_3(t)D_4(t) - B_4(t)D_3(t)] [A_2(t)C_1(t) - A_1(t)C_2(t)] \tag{23}
\end{aligned}$$

Here, the overdot means the time derivative. The expressions for the coefficients for the other coordinate are obtained from these expressions using the following replacements: $A_i \leftrightarrow B_i$ and $C_i \leftrightarrow D_i$ ($i = 1, 2, 3, 4$).

The equations for the second moments (variances),

$$\sigma_{q_1 q_j}(t) = \frac{1}{2} \langle q_i(t)q_j(t) + q_j(t)q_i(t) \rangle - \langle q_i(t)q_j(t) \rangle \tag{24}$$

where $q_i = R, \beta, P$ or P_β ($i = 1 - 4$), are

$$\begin{aligned}
\dot{\sigma}_{RR}(t) &= \frac{2\sigma_{RP}(t)}{m_1} \\
\dot{\sigma}_{\beta\beta}(t) &= \frac{2\sigma_{RP_\beta}(t)}{m_2} \\
\dot{\sigma}_{R\beta}(t) &= \frac{\sigma_{\beta P}(t)}{m_1} + \frac{\sigma_{RP_\beta}(t)}{m_2} \\
\dot{\sigma}_{RP_\beta}(t) &= -\lambda_{P_\beta}\sigma_{RP_\beta}(t) + \rho_\beta\sigma_{RP}(t) - c_\beta\sigma_{R\beta}(t) + \delta_\beta\sigma_{RR}(t) + \frac{\sigma_{PP_\beta}(t)}{m_1} + 2D_{RP_\beta}(t) \\
\dot{\sigma}_{RP}(t) &= -\lambda_P\sigma_{RP}(t) + \rho_R\sigma_{RP_\beta}(t) - c_R\sigma_{RR}(t) + \delta_R\sigma_{R\beta}(t) + \frac{\sigma_{PP}(t)}{m_1} + 2D_{RP}(t) \\
\dot{\sigma}_{\beta P}(t) &= -\lambda_P\sigma_{\beta P}(t) + \rho_R\sigma_{\beta P_\beta}(t) - c_R\sigma_{R\beta}(t) + \delta_R\sigma_{\beta\beta}(t) + \frac{\sigma_{PP_\beta}(t)}{m_2} + 2D_{\beta P}(t) \\
\dot{\sigma}_{\beta P_\beta}(t) &= -\lambda_{P_\beta}\sigma_{\beta P_\beta}(t) + \rho_\beta\sigma_{\beta P}(t) - c_\beta\sigma_{\beta\beta}(t) + \delta_\beta\sigma_{R\beta}(t) + \frac{\sigma_{P_\beta P_\beta}(t)}{m_2} + 2D_{\beta P_\beta}(t) \\
\dot{\sigma}_{PP_\beta}(t) &= -(\lambda_P + \lambda_{P_\beta})\sigma_{PP_\beta}(t) + \rho_R\sigma_{P_\beta P_\beta}(t) + \rho_\beta\sigma_{PP}(t) - c_R\sigma_{RP_\beta}(t) - c_\beta\sigma_{\beta P}(t) + \\
& + \delta_R\sigma_{\beta P_\beta}(t) + \delta_\beta\sigma_{RP}(t) + 2D_{PP_\beta}(t) \\
\dot{\sigma}_{P_\beta P_\beta}(t) &= -2\lambda_{P_\beta}\sigma_{P_\beta P_\beta}(t) + 2\rho_\beta\sigma_{PP_\beta}(t) - 2c_\beta\sigma_{\beta P_\beta}(t) + 2\delta_\beta\sigma_{RP_\beta}(t) + 2D_{P_\beta P_\beta}(t) \\
\dot{\sigma}_{PP}(t) &= -2\lambda_P\sigma_{PP}(t) + 2\rho_R\sigma_{PP_\beta}(t) - 2c_R\sigma_{RP}(t) + 2\delta_R\sigma_{\beta P}(t) + 2D_{PP}(t) \tag{25}
\end{aligned}$$

So we have obtained the Markovian-type (local in time) equations for the first and second moments, but with the transport coefficients depending explicitly on time. The time-dependent diffusion coefficients $D_{q_i q_j}(t)$ are determined as

$$\begin{aligned}
D_{RR}(t) &= -\frac{J_{RP}(t)}{m_1} + \frac{1}{2}\dot{J}_{RR}(t) \\
D_{\beta\beta}(t) &= -\frac{J_{\beta P_\beta}(t)}{m_2} + \frac{1}{2}\dot{J}_{\beta\beta}(t) \\
D_{R\beta}(t) &= -\frac{1}{2}\left[\frac{J_{\beta P}}{(t)}m_1 + \frac{J_{RP_\beta}(t)}{m_2} - \dot{J}_{R\beta}(t)\right] \\
D_{RP_\beta}(t) &= -\frac{1}{2}\left[-\lambda_{P_\beta}J_{RP_\beta}(t) + \rho_\beta J_{RP}(t) - c_\beta J_{R\beta}(t) + \delta_\beta J_{RR}(t) + \frac{J_{PP_\beta}(t)}{m_1} - \dot{J}_{RP_\beta}(t)\right] \\
D_{RP}(t) &= -\frac{1}{2}\left[-\lambda_P J_{RP}(t) + \rho_R J_{RP_\beta}(t) - c_R J_{RR}(t) + \delta_R J_{R\beta}(t) + \frac{J_{PP}(t)}{m_1} - \dot{J}_{RP}(t)\right] \\
D_{\beta P}(t) &= -\frac{1}{2}\left[-\lambda_P J_{\beta P}(t) + \rho_R J_{\beta P_\beta}(t) - c_R J_{RR}(t) + \delta_R J_{\beta\beta}(t) + \frac{J_{PP_\beta}(t)}{m_2} - \dot{J}_{\beta P}(t)\right] \\
D_{\beta P_\beta}(t) &= -\frac{1}{2}\left[-\lambda_{P_\beta} J_{\beta P_\beta}(t) + \rho_\beta J_{\beta P}(t) - c_\beta J_{\beta\beta}(t) + \delta_\beta J_{R\beta}(t) + \frac{J_{P_\beta P_\beta}(t)}{m_2} - \dot{J}_{\beta P_\beta}(t)\right] \\
D_{PP_\beta}(t) &= -\frac{1}{2}\left[-(\lambda_P + \lambda_{P_\beta})J_{PP_\beta}(t) + \rho_R J_{P_\beta P_\beta}(t) + \rho_\beta J_{PP}(t) - c_R J_{RP_\beta}(t) - c_\beta J_{\beta P}(t) + \right. \\
&\quad \left. + \delta_R J_{\beta P_\beta}(t) + \delta_\beta J_{RP}(t) - \dot{J}_{PP_\beta}(t)\right] \\
D_{P_\beta P_\beta}(t) &= \lambda_{P_\beta} J_{P_\beta P_\beta}(t) - \rho_\beta J_{PP_\beta}(t) + c_\beta J_{\beta P_\beta}(t) - \delta_\beta J_{RP_\beta}(t) + \frac{1}{2}\dot{J}_{P_\beta P_\beta}(t) \\
D_{PP}(t) &= \lambda_P J_{PP}(t) - \rho_R J_{PP_\beta}(t) + c_R J_{RP}(t) - \delta_R J_{\beta P}(t) + \frac{1}{2}\dot{J}_{PP}(t)
\end{aligned} \tag{26}$$

Here, $\dot{J}_{q_i q_j}(t) = dJ_{q_i q_j}(t)/dt$. In our treatment $D_{RR} = 0$, $D_{\beta\beta} = 0$, and $D_{R\beta} = 0$ because there are no random forces for the R and β coordinates in Eqs. (11). In Eqs. (26) we use the following notation:

$$\begin{aligned}
J_{RR}(t) &= \langle\langle I_R(t)I_R(t) + I'_R(t)I'_R(t)\rangle\rangle, \\
J_{\beta\beta}(t) &= \langle\langle I_\beta(t)I_\beta(t) + I'_\beta(t)I'_\beta(t)\rangle\rangle, \\
J_{PP}(t) &= \langle\langle I_P(t)I_P(t) + I'_P(t)I'_P(t)\rangle\rangle, \\
J_{P_\beta P_\beta}(t) &= \langle\langle I_{P_\beta}(t)I_{P_\beta}(t) + I'_{P_\beta}(t)I'_{P_\beta}(t)\rangle\rangle, \\
J_{PP_\beta}(t) &= \langle\langle I_P(t)I_{P_\beta}(t) + I'_P(t)I'_{P_\beta}(t)\rangle\rangle, \\
J_{R\beta}(t) &= \langle\langle I_R(t)I_\beta(t) + I'_R(t)I'_\beta(t)\rangle\rangle, \\
J_{RP}(t) &= \langle\langle I_R(t)I_P(t) + I'_R(t)I'_P(t)\rangle\rangle, \\
J_{\beta P_\beta}(t) &= \langle\langle I_\beta(t)I_{P_\beta}(t) + I'_\beta(t)I'_{P_\beta}(t)\rangle\rangle,
\end{aligned}$$

$$\begin{aligned}
J_{RP_\beta}(t) &= \langle \langle I_R(t)I_{P_\beta}(t) + I'_R(t)I'_{P_\beta}(t) \rangle \rangle, \\
J_{\beta P}(t) &= \langle \langle I_\beta(t)I_P(t) + I'_\beta(t)I'_P(t) \rangle \rangle.
\end{aligned}
\tag{27}$$

Thus, we obtain equations for the first and second moments with the transport coefficients explicitly depending on time, collective coordinate, and momentum. It is the time dependence of these coefficients that results from the non-Markovian nature of the system.

IV. CONCLUSIONS

A system of nonlinear Langevin equations is derived within the microscopic approach in the limit of the general coupling between the collective and internal subsystems. These equations of motion for the collective subsystem satisfy the quantum fluctuation-dissipation relations. A new method for obtaining explicitly time-dependent transport coefficients is developed on the basis of the non-Markovian Langevin equations. The analytical formulas obtained in this work can be used for describing the fluctuation-dissipation dynamics of nuclear processes.

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